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ALL INDIA TEST SERIES CSE-2023

Candidate 's Information

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4. **SUBJECT:-** Paper 2 Full Length test
5. **DATE:-** 27-08-23

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Q.NO	MARKS
1.	24
2.	
3.	28
4.	
5.	29½
6.	32
7.	30½
8.	

Very Good

TOTAL MARKS	144

250

EXAMINER SIGNATURE

INVIGILATOR SIGNATURE

AITS 2023 : Test 5 (Paper 2 FLT)

Q1 (a)

Bohr's correspondence principle states that for a particle, energy states are quantized i.e. he gave the concept of stationary states

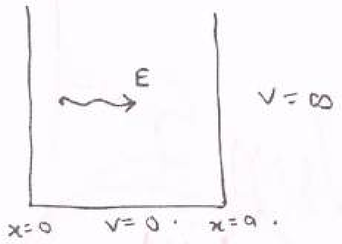


Fig 1. e^- in box

For electron in a box,

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$$

$$\Rightarrow \frac{d^2\psi}{dx^2} = -k^2\psi \quad \Big\} \; k = \frac{\sqrt{2mE}}{\hbar}$$

$$\Rightarrow \psi = A\sin kx + B\cos kx \, .$$

Boundary condition: $\psi\Big]_{x=0} = 0 \Rightarrow B = 0$

$\Rightarrow \psi = A\sin kx = 0$ at $x = a$

$\Rightarrow ka = n\pi$. Substituting value of k,

$$E = \frac{n^2\pi^2\hbar^2}{2ma^2} \Rightarrow \boxed{E = n^2\,(\text{constant})}$$

Hence, the states formed are stationary in line with Bohr's model.

Q1 (b)

De-Broglie's Wave-Particle Duality states that every moving particles behaves like a wave with wavelength $(\lambda) = h/p$.

(i) $V = 10\,V$ i.e. non relativistic regime as $E < m_0c^2$.

$$\Rightarrow E = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mE} \Rightarrow \lambda = \frac{h}{\sqrt{2mE}}$$

Substituting values, $\boxed{\lambda = 3.88\,\text{Å}}$

(ii) $V = 1\,MeV$ as $E > m_0c^2$, relativistic.

$$\Rightarrow E^2 = p^2c^2 + (m_0c^2)^2 \Rightarrow (pc)^2 = 1 - (0.51)^2$$

$$\Rightarrow pc = 0.86\,MeV$$

$$\Rightarrow \lambda = \frac{hc}{pc} \Rightarrow \text{wavelength} = \boxed{1.44 \times 10^{-12}\,m}$$

6/10

As seen, in relativistic regime, momentum increases which leads to drop in wavelength.

Q1 (c)

Zeeman splitting is splitting of j^{th} energy level into $(2j+1)$ sublevels in the presence of weak magnetic field.

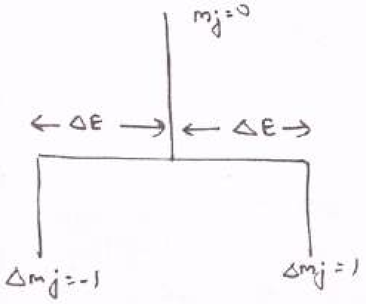


Fig 1. Components in normal Zeeman

> In normal Zeeman,

$$S = 0 \Rightarrow J = L$$

$$\Rightarrow g_J = 1$$

> Energy of splitting

$$E = -\vec{\mu_J} \cdot \vec{B}$$

$$\Rightarrow \Delta E = \frac{e}{2m} g_J m_J \hbar B$$

$$\Rightarrow \Delta E = \mu_B B \text{ . Also } \Delta E = d\left(\frac{hc}{\lambda}\right) = \frac{hc\Delta\lambda}{\lambda^2}.$$

$$\Rightarrow \Delta\lambda = \frac{(\mu_B B)(\lambda^2)}{hc} = \underline{0.33 \text{ A}^\circ}.$$

∴ Wavelengths of components are λ, $\lambda + \Delta\lambda$ & $\lambda - \Delta\lambda$ i.e. $\boxed{4226.4 \text{A}^\circ}$, $\boxed{4226.73 \text{A}^\circ}$ & $\boxed{4227.06 \text{A}^\circ}$. The separation between them would be $\boxed{0.33 \text{A}^\circ}$.

Zeeman effect is used in laser cooling application.

6½/10

Q.1 (d)

Postulates of quantum mechanics state that every measurable quantity can be represented by a wave function probability

(i) Expectation value of any physical Weighted by quantity is the average value that would be obtained if it is measured multiple times.

> If the wave function is Ψ, expectation value of any quantity A is $\langle \Psi | A | \Psi \rangle$.

By $\langle A \rangle$, we can know,

(i) Shape of wave function

(ii) Error in measurement if $\langle A^2 \rangle$ known.

5/10

$\boxed{(ii)}$. Given:- $\Psi = ax$ for $0 < x < 1$

We know, probability $= \Psi\Psi^*$. As particle will be found somewhere,

$$\langle \Psi | \Psi^* \rangle = 1 \Rightarrow \int_{-\infty}^{0} \underset{0}{\Psi^*\Psi} + \int_{0}^{1} \Psi\Psi^* + \int_{1}^{\infty} \underset{0}{\Psi^*\Psi} = 1$$

$$\Rightarrow \boxed{\int \Psi\Psi^* = 1} \quad \therefore \boxed{\text{Prob } (x=0 \text{ to } x=1) = 1}.$$

Q1(e)

Potential energy is the energy of electron due to coulombic interaction with proton.

We know, $V = \frac{-e^2}{4\pi\epsilon_0 r} \Rightarrow \langle v \rangle = \frac{-e^2}{4\pi\epsilon_0}\left\langle \frac{1}{r} \right\rangle$

$$\Rightarrow \left\langle \frac{1}{r} \right\rangle = \int_0^\infty \frac{1}{r}\cdot\frac{1}{\pi a_0^3} e^{-2r/a_0}\cdot 4\pi r^2 \, dr .$$

$$\Rightarrow \left\langle \frac{1}{r} \right\rangle = \frac{4}{a_0^3}\int_0^\infty r e^{-2r/a_0} \, dr \quad \Bigg\} \begin{array}{l} \text{Let } \frac{2r}{a_0} = x \\ dr = \frac{a_0}{2} dx \end{array}$$

$$\Rightarrow \left\langle \frac{1}{r} \right\rangle = \frac{4}{a_0^3}\cdot\left(\frac{a_0}{2}\right)^2 \underbrace{\int_0^\infty x e^{-x} \, dx}_{1}$$

$$\Rightarrow \left\langle \frac{1}{r} \right\rangle = \frac{1}{a_0} = (0.529 A^\circ)^{-1}$$

$$\Rightarrow \langle v \rangle = \frac{-e^2}{4\pi\epsilon_0}\times \underbrace{\frac{me^2}{\hbar^2}}_{\text{Given}} = \frac{-me^4}{4\epsilon_0\hbar^2}$$

6½ / 10

Substituting values, $\langle v \rangle = \boxed{-27.2\,eV}$

As seen, the potential energy is twice of total energy.

Q3 (a).

Particle in potential well is characterized by potential $\propto x^2$ where r is the distance. Hence, Schrodinger wave Eq. is

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+\left(\frac{1}{2}kx^2\right)\psi = E\psi .$$

But we define 2 operators as :-

$$a_+ = \text{Raising} = \frac{1}{\sqrt{2m\hbar w}}\left[mwx - iP_x\right]$$

$$a_- = \text{Lowering} = \frac{1}{\sqrt{2m\hbar w}}\left[mwx + iP_x\right]$$

~~Let~~ The operator operate as :-

$$a_+|n\rangle = \sqrt{n+1}\,|n+1\rangle \;\&\; a_-|n\rangle = \sqrt{n}\,|n-1\rangle .$$

for state n. Let the ground state wave function be ψ_0.

$$\Rightarrow a_-|\psi_0\rangle = 0 .$$

$$\Rightarrow mwx\psi_0 + i\left(-i\hbar\frac{\partial\psi_0}{\partial x}\right) = 0 .$$

$$\Rightarrow mwx\psi_0 = -\hbar\frac{\partial\psi_0}{\partial x}$$

$$\Rightarrow \int\frac{\partial\psi_0}{\psi_0} = \int\frac{mw}{-\hbar}x\,dx \Rightarrow \psi_0 = Ae^{-\frac{mw}{2\hbar}x^2}$$

We know, $\langle \psi_0 | \psi_0 \rangle = 1$

$$\Rightarrow \int_{-\infty}^{\infty} |A| \, e^{-\frac{m\omega}{\hbar}x^2} \, dx = 1$$, on solving,

$$\Rightarrow |A| = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} .$$

$$\Rightarrow \boxed{\psi_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2}} .$$

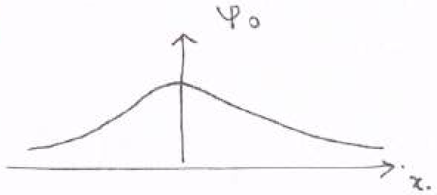


Fig 1. ψ_0 state.

Also, $a_+ a_- = \frac{H}{\hbar\omega} - \frac{1}{2}$. We know, $H\Psi = E\Psi$

$$\Rightarrow \frac{H}{\hbar\omega}|\psi_0\rangle = a_+ a_- + \frac{1}{2}|\psi_0\rangle$$

$$\Rightarrow \frac{E}{\hbar\omega}\psi_0 = \frac{\psi_0}{2} \Rightarrow \boxed{E = \frac{\hbar\omega}{2}} .$$

The quantum oscillator differs from classical as $E_{min} \neq 0$, energy levels are quantized & probability of finding, particle outside of max. amplitude ~ 16%.

13/20

Q3 (b).

Particle in a finite well can be modeled using Schrodinger Wave Equation which gives symmetric & antisymmetric solutions -

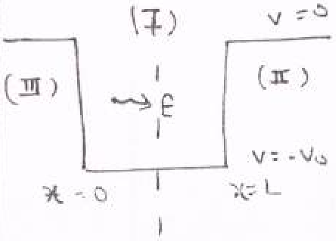


Fig 1. Particle in finite well

> We know, for region (I)

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = E\psi$$

$$\Rightarrow \psi_I = A\sin k_1 x + B\cos k_1 x$$

where $k_1 = \frac{\sqrt{2m(V_0-E)}}{\hbar}$

$\psi_{II} = Ce^{-k_2 x}$; $k_2 = \frac{\sqrt{2mE}}{\hbar}$

Boundary conditions :- $\psi_I = \psi_{II}]_{x=L}$ & $\partial\psi_I/\partial x = \partial\psi_{II}/\partial x]_{x=L}$ .. Here

2 cases arise :

① Anti-symmetric solutions :- $\psi_I = A\sin k_1 x$

$$\Rightarrow A\sin k_1 L = Ce^{-k_2 L}$$

$$-k_1 A \overset{\cos}{\cancel{\sin}} k_1 L = k_2 e^{-k_2 L}.$$

$$\Rightarrow x\tan\left(\frac{\pi}{2}+x\right) = y$$

where, $x = k_1 L$
$y = k_2 L$.

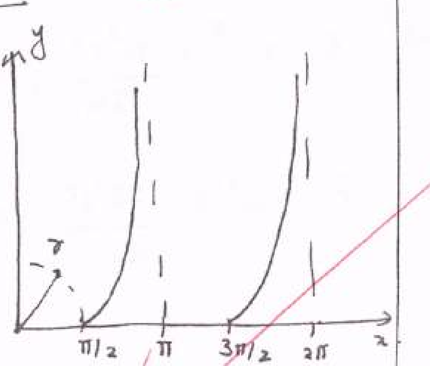


Fig 2. Anti-symmetric

⇒ If $x^2+y^2=r^2$, For $r<(n-1/2)\pi$,

We have, $(n-1)$ Anti-symmetric states.

② Symmetric States :- $\psi_I = B\cos k_1 r$.

⇒ $B\cos k_1 L = Ce^{-k_2 L}$.

$Bk_1\sin k_1 L = Ck_2 e^{-k_2 L}$.

⇒ $x\tan x = y$

where $x = k_1 L$

$y = k_2 L$.

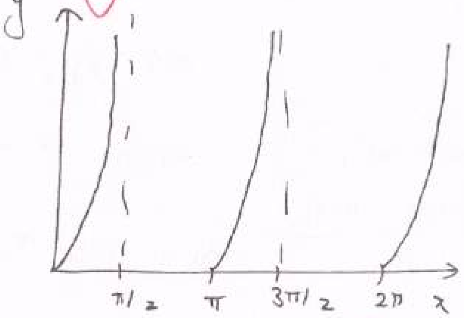


Fig 3. Symmetric states

⇒ For $x^2+y^2=r^2$

If $r<(n-1/2)\pi$,

n symmetric bound-states.

Now, $x^2+y^2 = \frac{2mV_0}{\hbar^2}\cdot L^2 = \frac{2m}{\hbar^2}(V_0L^2)$

Now if $V_0L^2 = \frac{\pi^2\hbar^2}{8m}$, $r^2 = \frac{\pi^2}{4} \Rightarrow r = \frac{\pi}{2}$

if $V_0L^2 = \frac{4\pi^2\hbar^2}{2m}$, $r^2 = 4\pi^2 \Rightarrow r = 2\pi$

$\therefore$ For $\frac{\pi}{2} < \gamma \leq 2\pi$, we will have,

$\boxed{2 \text{ antisymmetric}}$ & $\boxed{3 \text{ symmetric}}$ state

$\Rightarrow$ Total no. of bound states = $\boxed{5}$.

Particle in finite well is used to model deuteron to estimate range of nuclear force.

$\frac{10}{20}$

$\gamma^2 = \frac{mV_0L^2}{2\hbar^2}$

$\frac{\pi}{4} < \gamma \leq \pi$

Only 3 solns.

Q3(c)

· Raman Spectra is observed when scattered light gives frequency both greater & less than incident light.

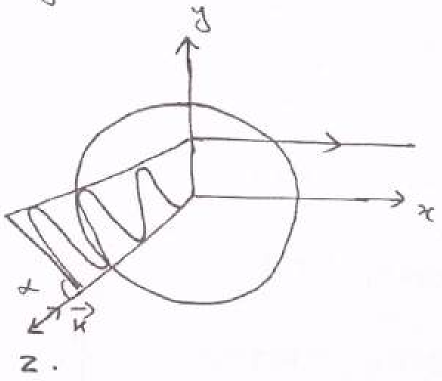


Fig 1. Light incident on molecule

> Consider a light with plane of polarization at ∠α with z-axis falling on spherical molecule like CH_4.

> As the molecule is symmetric, its sphere of polarizability will expand equally when light falls on it.

> Its bonds will vibrate & in plane of incident light.

> As light emitted is ⊥ to plane of vibration, the plane would always be xy plane irrespective of ∠α.

> However, not all molecules are spherically symmetric. In such cases,

polarization would be maximum along minor axis of ellipsoid. & the light would be polarized to different extents.

> Hence, light will always lie in x-y plane (to different extent) & will be plane polarized.

Degree > polarizability

$r = \frac{I_\perp}{I_{\parallel}}$

Raman spectra is used to analyze molecules without centre of symmetry.

5/10

Q5(a).

Nuclear properties are the properties of nucleus i.e. a region where protons & neutrons exist.

① Volume $\propto A$:- Volume of nucleus is proportional to its mass no. (A). This is because, radius of nucleus $R = R_0 A^{1/3}$

$$\Rightarrow \text{Volume} = 4/3 \pi R^3 = \left(\frac{4}{3}\pi R_0^3\right) \underline{A}$$

② Constant density :- As mass of nucleus is $\approx A$ a.m.u & volume $\propto A$,

$$\rho = \frac{A\,(\text{a.m.u})}{4/3\pi R_0^3 A} \approx \underline{10^{17}\ kg/m^3} \text{ i.e.}$$

density of nucleus across atoms is constant.

Numerical For ^{12}C, $A = 12$

$$\Rightarrow R = R_0 A^{1/3} = \boxed{2.75\ fm}$$

$$\frac{\text{Atomic radius}}{\text{Nuclear radius}} = \frac{0.529 \times 10^{-10}}{2.75 \times 10^{-15}} = \underline{1.9 \times 10^4}$$

∴ Atomic radius of carbon is $\sim 10^4$ times that of nuclear radius.

The inference that can be drawn is that nuclear ~~range~~ force is very short range i.e. 10^{-15} m. However, coulombic force i.e. e–p force has longer range.

6/10

Q.5 (b)

The number of neutrons v/s number of protons plot can help estimate relative Stability of nucleus.

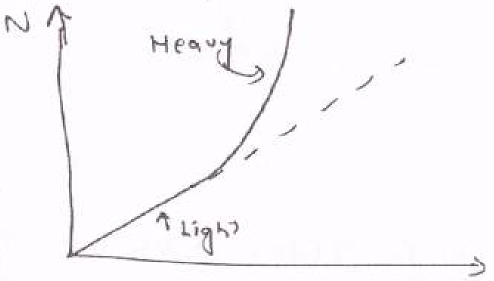


Fig 1. N v/s Z plot

(i) Inference

① For lighter nucleii, $Z < 20$, $N = Z$.

② For heavier nucleii, $N > Z$. because coulombic repulsion increases more rapidly v/s nuclear attraction ⇒ More neutrons needed to balance repulsion.

$5\frac{1}{2}$ / 10

(ii) Between $^{7}_{3}$Li & $^{8}_{3}$Li, $^{7}_{3}$Li would be more stable as N & Z are closer. For $^{8}_{3}$Li, nuclear Forces will dominate the coulombic force leading to atom collapse.

This Z to N conversion gives the asymmety term in Weizsacker-Bethe Formula.

Q5 (c).

Cell is a fundamental unit of structure replicating whom along translation vector gives the whole crystal. Depending on placement of atoms, we have 3 types:-

① Simple Cubic.

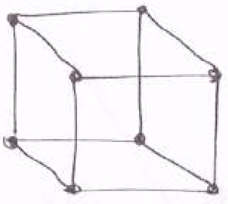


Fig1. Simple Cubic.

Here, $2r = a$

$\Rightarrow$ # atoms / cell $= \boxed{1}$

$\Rightarrow$ Packing fraction $= \dfrac{(1)(4/3\pi r^3)}{a^3}$

$= \boxed{52\%}$.

Example:- Polonium.

② Body-Centered Cubic.

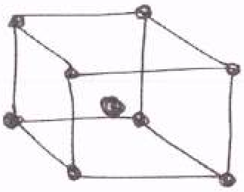


Fig 2. BCC Cell.

Here, atoms touch along body diagonal

$\Longrightarrow \sqrt{3}a = 4r$.

$\Longrightarrow$ # atoms / cell $= \boxed{2}$

$\Longrightarrow$ Packing fraction $= \dfrac{2 \times 4/3\pi r^3}{a^3}$

$= \boxed{68\%}$

6

(1)

Example:- Na, K.

③ **Face - centered Cubic**

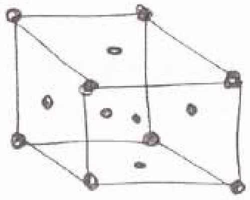


Fig 3. Fcc cell

> Atoms touch along face diagonal $\Rightarrow 4r = \sqrt{2}a$.

$\Rightarrow$ # atoms / cell $= \boxed{4}$

$$\Rightarrow P.f = \frac{4\left(\frac{4}{3}\pi r^3\right)}{a^3} = \boxed{74\%}$$

Example :- Cu, Ag.

Simple Cubic & BCC are called loose packed structure due to low packing fraction while FCC is closed packed structure.

Q.5 (d)

Bound-state is a state for a particle when it is confined to particular region due to some potential.

> n-n scattering establishes that both di-proton & di-neutron do not exist.

> Can be explained via exchange force model :-

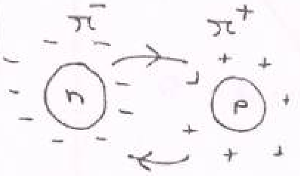


Fig 1. Exchage forces

> Particles like neutron & proton are attracted due to exchage of $\pi^{\pm}$ mesons. This leads to bound-state.

> In case of neutron-neutron, no particles will be exchanged ⇒ No force.

> To account for it, Iso-spin(T) has been introduced with $T_z = 1/2$ for proton & $T_z = -1/2$ for neutron

This shows that nuclear force follows Iso-spin symmetry.

5/10

Q.5(e)

Gellman - Newmann proposed quark model of Hadrons stating that each hadron can be written as function of quarks.

The quarks & properties are:-

Quark	Charge	B	Spin	Tz	Strangeness
u	$2/3\,e$	$1/3$	$1/2$	$1/2$	0
d	$-1/3\,e$	$1/3$	$1/2$	$-1/2$	0
s	$-1/3\,e$	$1/3$	$1/2$	0	-1

(i) π^0 :- Q = 0, B = 0, Tz = 0, Strange = 0

$$\Rightarrow \boxed{\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})}$$

(ii) k^+ :- Q = 1, B = 0, Tz = 1/2, Strange = 1

$$\Rightarrow \boxed{k^+ = u\bar{s}}$$

(iii) Δ^{++} :- Q = 2, B = 1, Tz = 3/2, Strange = 0.

$$\Rightarrow \boxed{\Delta^{++} = uuu}$$

(iii) Σ^0 :- Q = 0, B = 1, Tz = 0, Strange = −1

$$\Rightarrow \boxed{\Sigma^0 = uds}$$

This model is further used to study decay of strange particles.

6½/10

Q 6 (a) (i)

Bethe - Weizsacker gave semi-emperical mass Formula to estimate mass & binding energy of atoms as:-

$$M(Z,A) = ZM_p + (A-Z)M_n - \underbrace{a_1 A}_{E_v} + \underbrace{a_2 A^{2/3}}_{E_s}$$

$$+ \underbrace{a_3 \frac{Z(Z-1)}{A^{1/3}}}_{E_c} + \underbrace{a_4 \frac{(A-2Z)^2}{A}}_{E_a} \underset{0}{\pm} \underbrace{a_5 A^{-3/4}}_{E_p} .$$

① Volume energy (E_v): Accounts of nuclear force & is $\propto V \Rightarrow E_v \propto \frac{4}{3}\pi(R_0 A^{1/3})^3$

$\Rightarrow E_v = -a_1 A$ } As it is attractive.

② Surface energy (E_s): Nucelons at surface interact with lesser no. of nucleons $\Rightarrow$ E decreases

$\therefore E_s \propto$ surface area $\Rightarrow E_s = a_2 A^{2/3}$

③ Coulombic energy (E_c): Accounts for proton - proton interaction $E_c = \frac{Z c_2 e^2}{4\pi\varepsilon_0 \langle r_{ij} \rangle}$

$\Rightarrow E_c = a_3 \frac{Z(Z-1)}{A^{1/3}}$

④ Asymmetry energy (E_a): At higher mass no, no. of neutrons > Z to account for.

increased repulsion. Also some protons convert into neutrons. For every $\left(\frac{N-Z}{2}\right)$ conversions, we have

$\Delta E = \left(\frac{N-Z}{2}\right)^2 (a)$

$\Rightarrow E_a = a_4 \frac{(N-Z)^2}{A}$

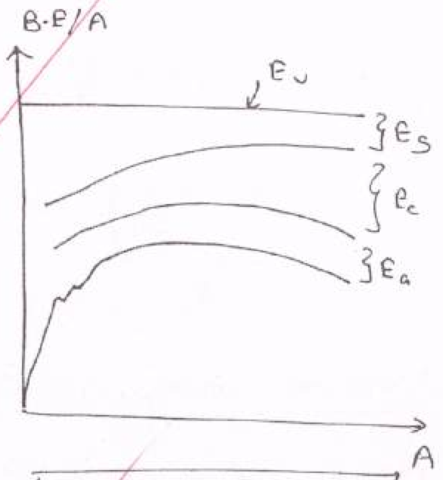


Fig 1. BE/A v/s A

⑤ Pairing energy (E_p):-

The stability shown by nucleus is N-Z as even-even > even-odd > odd-even > odd-odd

Hence E_p = + for ~~odd~~ even-even
− for oddodd nucleus
0 for odd A nucleus.

(ii) Exchange of pions leads to attractive force which is responsible for nuclear force.

> From Heisenberg's uncertainty principle, we know, $\Delta E \cdot \Delta t \geq \hbar/2$.

Now $\Delta E \sim E = m_\pi c^2$, $\Delta t = \frac{Range}{c} = \frac{R}{c}$.

$\Rightarrow (m_\pi c^2) \cdot \frac{R}{c} \geq \frac{\hbar}{2}$

$\Rightarrow R \geq \frac{\hbar}{2 m_\pi c}$. Substituting values,

we get range = $\boxed{0.7 \text{ fm}}$.

$\therefore$ Range of nuclear force is $\boxed{\sim 1\text{fm}}$ which is also the size of nucleus.

7½

15

Q. 6 (b)

Shell - model analyzes one nucleon in potential of (A-1) nucleons. One of this type of potential is harmonic potential.

$\Rightarrow$ $-\frac{\hbar^2}{2m}\nabla^2\psi + \left(\frac{1}{2}kr^2\right)\psi = E\psi$. On solving,

we get, $E = (N + 3/2)\hbar\omega$

where $N = 2n + l - 2$. Here $N = 0, 1, 2 \ldots$

$l = 0, 1, 2 \ldots$

$n = 0, 1, 2 \ldots$

Solving for values of N :-

N	n , l	State	# Nucleons
0	n = 1 , l = 0	1S	2
1	n = 1 , l = 1	1P	6
2	n = 1 , l = 2	1D	10
	n = 2 , l = 0	2S	2
3	n = 2 , l = 1	2P	6
	n = 1 , l = 3	1F	14

⋮

As seen, for N = 1, we have 6 degenerate states, for N = 2 we have 12 & so on. In general, for N, we have $(N+1) \times (N+2)$ degenerate states for different values of n & l. Hence, there is large degeneracy for particular N.

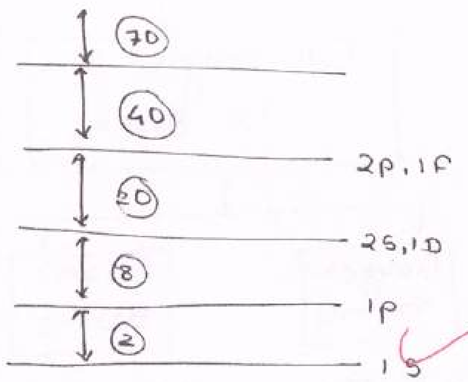


Fig1. Magic numbers predicted

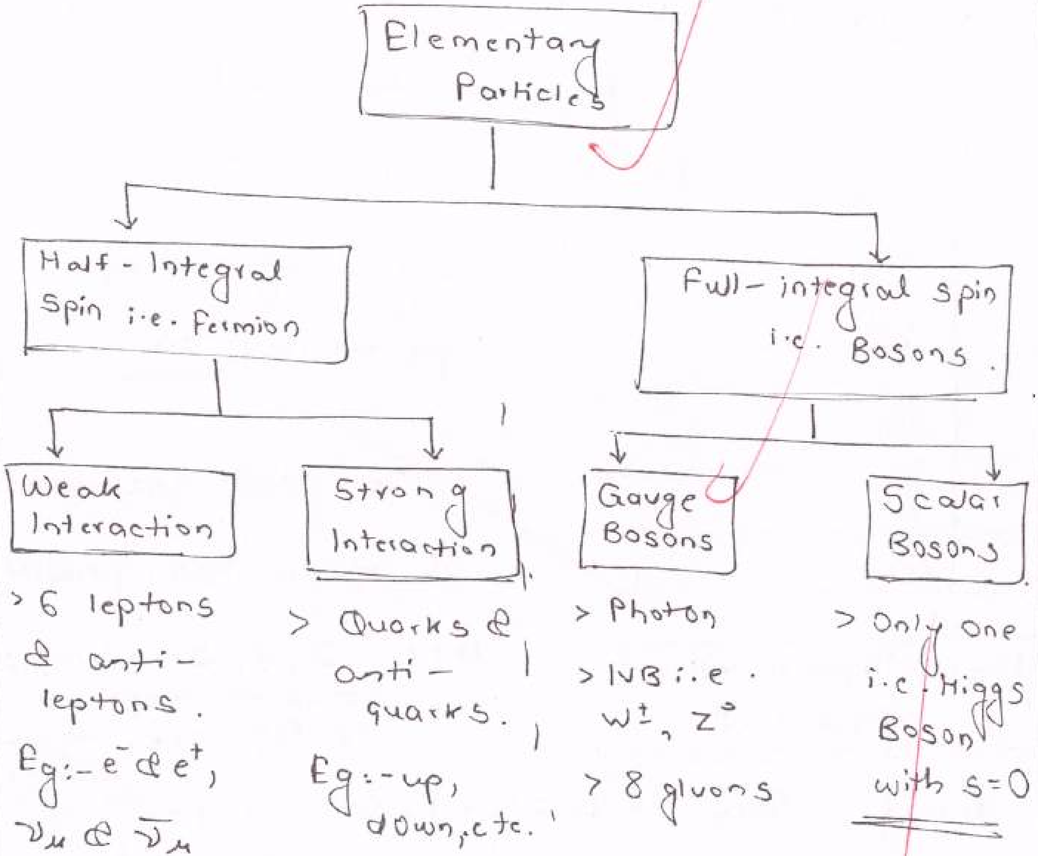
> The Shell enclosures & magic nos. predicted are 2,8,20,40,70 .... → As seen they diverge from 2.8,20,28,50... for higher magic numbers

9½
15

Mayer in 1949 introduced a non-central potential in the form of spin-orbit which led to splitting of I^{th} level to match magic numbers.

Q.6 (c)

Elementary particles are sub-atomic particles which cannot be divided further.



- Elementary Particles
  - Half-Integral Spin i.e. Fermion
    - Weak Interaction
      - > 6 leptons & anti-leptons.
      - Eg:- e^- & e^+, ν_μ & $\bar{\nu}_\mu$
    - Strong Interaction
      - > Quarks & anti-quarks.
      - Eg:- up, down, etc.
  - Full-integral spin i.e. Bosons.
    - Gauge Bosons
      - > Photon
      - > IVB i.e. $W^\pm$, Z^0
      - > 8 gluons
    - Scalar Bosons
      - > Only one i.e. Higgs Boson with s=0

Other types of classification

① Strangeness (Gellman - Nishijima).

Strangeness = 0 . Eg:- Leptons, π, etc.

Strangeness $\neq 0$ Eg:- k^+, Σ^0, etc. i.e. strange particles.

② Mass :- Leptons (e^-, ν_e, etc.) < Mesons (π, k, η, etc.) < Boryons (n, p, Λ, Ξ, etc.)

These elementory particles interact through various ways as :-

Interaction	τ	Range	Particle	Exchange Particle	Strength
Strong	10^{-23} s	10^{-15} m	Hadrons	Gluons	1
Electro-magnetic	$< 10^{-14}$ s	∞	Charged particles	γ	10^{-4}
Weak	$> 10^{-8}$ s	10^{-17}	Leptons	$W^{\pm}$, Z^0	10^{-14}
Gravity	—	∞	Particle with mass	Graviton	10^{-40}

The properties of these interactions are :-

① Strong :- Short range, attractive, charge, mass independent, tensor.

② EM :- Both attractive, repulsive.

③ Weak :- Shortest range, frame variant, seen in strange particle decay.

(4) Gravity :- Always attractive but weakest.

Although standard model of physics assumes these to be different interactions, Grand Unification Theory states that they are manifestation of same force & would unify at 10^{15} GeV.

Till now, electro-weak unification has been achieved at 10^{2} GeV. Detection of Higgs Boson in 2012 has given new impetus to GUT.

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Q.7 (a) (i)

Binding energy is the energy released when constituent nucleons come together in a nucleus.

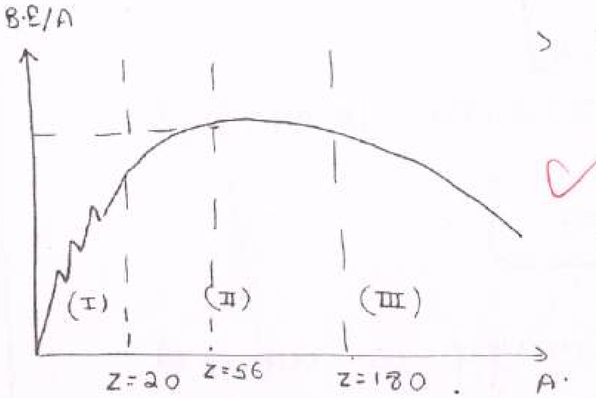

Fig1. B.E/A v/s A

- B.E v/s A is an inverted parabola.
- Every nucleus tries to achieve more stability by increasing its Binding energy (BE)
- If Z < 20, atoms are lighter → Combine to increase BE ⇒ ~~Fission~~ Fusion. Eg:- ^1H
- If Z > 180, atoms are very heavy → Break into constituent to decrease B.E. ⇒ fission. Eg:- ^{235}U.
- Region (II) :- Nucleus are usually stable & don't show ~~radio~~ nuclear fission / fusion

Eg:- ^{56}Fe has highest BE & is very stable.

(ii) The energy of nuclear reaction can be given by the Q-value. We know,

$Q = (m_i - m_f) c^2$.

$\Rightarrow Q = (12 + 4.002603 - 15.999) c^2$

$\Rightarrow \boxed{Q = 3.38 \text{ MeV}}$

As energy is released in each reaction, eventually star runs out of fuel to form white dwarf, neutron star or black hole.

6½/10

Q.7 (b)

Josephson effect is tunnelling of cooper pairs via thin insulator (~10Å) sandwiched between two identical superconductors

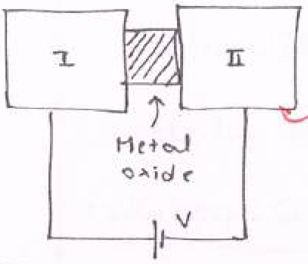

Fig1. Josephson junction

Here, 2 cases obtained:-

(A) DC Josephson i.e. $V=0$

Even in this case, cooper pairs tunnel through

(B) AC Josephson i.e. $V=V_0$.

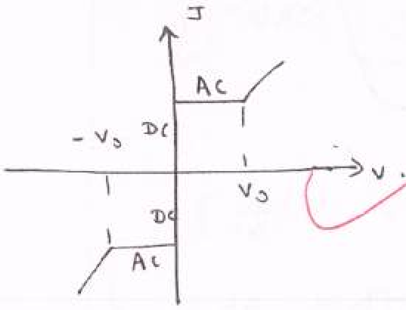

Fig 2. AC & DC Josephson.

We know, energy of cooper pairs $= 2eV_0$

Now $2eV_0 = h\nu$

$\Rightarrow \nu = \frac{2eV_0}{\hbar}$. ∴ If phase difference at $t=0$ is $\delta_{(0)}$,

We have current across Josephson as $J = J_0 \sin\left(\delta_{(0)} \pm \frac{eV_0}{\hbar} t\right)$

This effect is further used to construct SQUID (Superconducting Quantum Interference Device) which are 2 Josephson connected in parallel.

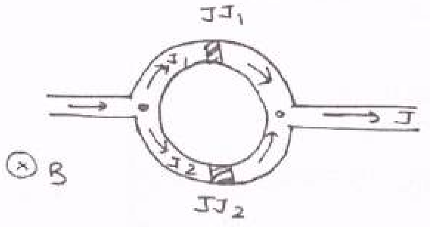

Fig 3. SQUID

> Magnetic Quantum Interference states that when 2 parallel Josephson junctions are kept in constant magnetic field, the super-curient interferes leading to maxima & minima.

> Experimentally, we know, $\Phi = s\left(\frac{h}{2e}\right)$

$\Rightarrow 2\pi s = \frac{2e\Phi}{\hbar}$. We split this as :-

$$J_1 = J_0 \sin\left(\delta_0 - \frac{e\Phi}{\hbar}\right) \;\&\; J_2 = J_0 \sin\left(S_0 + \frac{e\Phi}{\hbar}\right)$$

$$\Rightarrow J = J_1 + J_2 = 2J_0 \cos\left(\frac{e\Phi}{\hbar}\right) \sin(S_0)$$

⇒ Every time $\frac{c\Phi}{\hbar} = \pi$, we get a maxima

> We measure this maxima using galvanometer → find Φ. If we know area, we can find magnetic field (B).

Applications of JJ & SQUID

① Josphson junction used to measure extremely small voltages i.e. μV as $V \propto \nu(Hz)$ which can be easily measured

② SQUID is used to measure extremely small magnetic fields like magnetic field at centre of atom due to rotating electron.

$\frac{12\frac{1}{2}}{20}$

Q 7 (c)

Junction Field Effect Transistor (JFET) is a type of transistor in which conduction happens only via one type of carrier i.e. electron or hole.

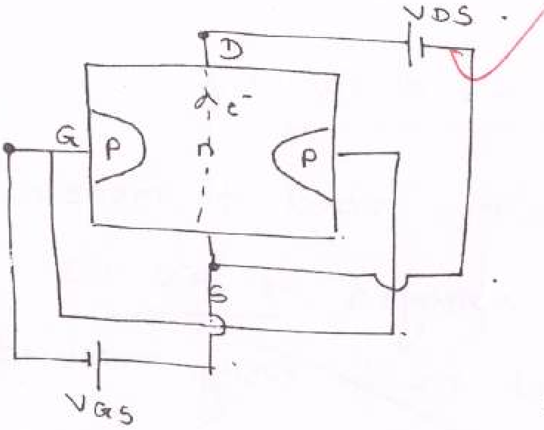

Fig1. n-channel JFET

- In n-channel JFET, we have 2 regions doped with p-type while the channel is n-type.
- It conducts through flow of electrons.
- The n-p boundary has a depletion region.
- If $V_{GS} < 0$, as V_{GS} increases, the depletion region decreases. ⟹ Flow of e^- from Source to drain decreases. ⟹ we get voltage controlled resistor.

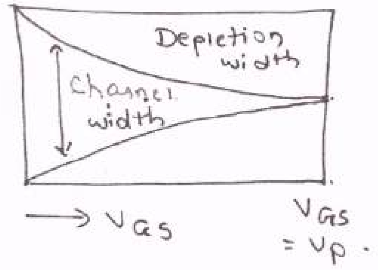

Fig 2. Variation of channel with V_{GS}

> Pinch-off voltage V_p = Voltage at which drain current $(I_D) = 0$ i.e. depletion region becomes so large that it completely closes the n-channel.

> The voltages are related as :-

$$I_D = I_{DSS}\left[1 - \frac{V_{GS}}{V_P}\right]^2$$ where

V_{GS} = Gate-source voltage.
I_{DSS} = Saturation current.
V_P = Pinch-off voltage.
I_D = Drain current.

$11\frac{1}{2}$ / 20

Advantages of JFET v/s BJT :- ① Higher input impedance ⇒ Low noise ② High power gain ⇒ No need of driver circuit.

Usually, IC 2N547 is used as JFET.